**Set 2 Part 1 Test Cases**

**General Instructions (same as Set 1)**

- Please provide mean hazard results (probability of exceedance) for peak horizontal acceleration (PGA) defined at 0.001, 0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7, 0.8, 0.9, and 1.0 g. (*Note Test Case 2.5 has different PGA values specified)
- Assume a Poisson model when converting rates to annual probabilities of exceedance
- Use 16.05 (not 16.1) in the equation $\log M_o = 16.05 + 1.5\mathbf{M}$
- Use $3 \times 10^{11}$ dyne/cm$^2$
- Use a magnitude integration step size small enough to define the specified magnitude density function. The bin size for magnitude integration should be defined such that the $M_{min}$ is at the lower edge of a bin, not in the center (i.e., If your magnitude step size is 0.01, one magnitude bin should be from **M** 5.0 to 5.01)
- When integrating over the magnitude density function (balancing the moment), integrate from zero (not $M_{min}$)
- Sigma = 0 for the ground motion model implies that the sigma in the relationship is artificially set to zero, not that the sigma is truncated
- Note that equation for ln(y) in Table 3.1 of Sadigh et al. (1997) has a typo in the third term. It should read C3*(8.5-M)^2.5 to match equation 2.2.
- Rupture dimension relationships:

  Log (A) = **M** – 4 $\sigma_A = 0$

  Log (W) = (0.5***M**) – 2.15 $\sigma_W = 0$

  Log (L) = (0.5***M**) – 1.85 $\sigma_L = 0$

  ($\sigma_A = \sigma_W = \sigma_L = 0$ for all test cases in this set)

  Aspect Ratio (L/W) = 2
- Maintain the aspect ratio defined until maximum width is reached, then increase length (conservation of area at the expense of aspect ratio)
- Rupture plane location is uniformly distributed along strike and down dip. Do not allow rupture off the ends of fault. This results in uniform slip with tapered edges. Downdip and along-strike integration step size should be small enough to produce uniform rupture location. (*Note Test Case 2.4b has a different distribution specified downdip)
- You should be using as small a step size as feasible/necessary to produce stable results (for both magnitude density function and rupture distribution). This may be much smaller than you normally use on projects.

**Case 2.1 – Multiple sources, Deaggregation**

Description: Calculate the hazard for Site 1 shown in Figure 2.1 due to the area source, Fault B, and Fault C.

Area Source: circle with r = 100 km, point source depths = 5-10 km, $N(M \geq 5) = 0.0395$, use 1 km grid spacing of point sources or small faults on the horizontal plane to simulate uniform distribution (for depth distribution use 1 km spacing inclusive of 5 and 10 km, so there are 6 depths - 5, 6, 7, 8, 9, and 10 km, equal weighting)
Magnitude Density Function: truncated exponential model, $M_{min} = 5.0$, $M_{max} = 6.5$, b-value = 0.9

Fault B: L = 85 km, fault plane depths = 0-12 km, strike-slip, dip 90°, slip rate = 2 mm/yr
Magnitude Density Function: characteristic model (Youngs and Coppersmith [1985]), b-value = 0.9, $M_{min} = 5.0$, $M_{char} = 6.75$, $M_{max} = 7.0$

Fault C: L = 50 km, fault plane depths = 0-12 km, strike-slip, dip 90°, slip rate = 1 mm/yr
Magnitude Density Function: characteristic model (Youngs and Coppersmith [1985]), b-value = 0.9, $M_{min} = 5.0$, $M_{char} = 6.5$, $M_{max} = 6.75$

Ground motion model: Sadigh *et al*. (1997), rock, σ untruncated

Rupture plane (for faults):

$\text{Log}(A) = \mathbf{M} - 4$  $\sigma_A = 0$
$\text{Log}(W) = (0.5 * \mathbf{M}) - 2.15$  $\sigma_W = 0$
$\text{Log}(L) = (0.5 * \mathbf{M}) - 1.85$  $\sigma_L = 0$
Aspect Ratio (L/W) = 2
Maintain the aspect ratio defined until maximum width is reached, then increase length (conservation of area at the expense of aspect ratio)
Uniform distribution along strike and down dip

**Case 2.1 – Multiple sources, Deaggregation (continued)**

Deaggregation: Provide deaggregation results for the following three PGA values at Site 1:

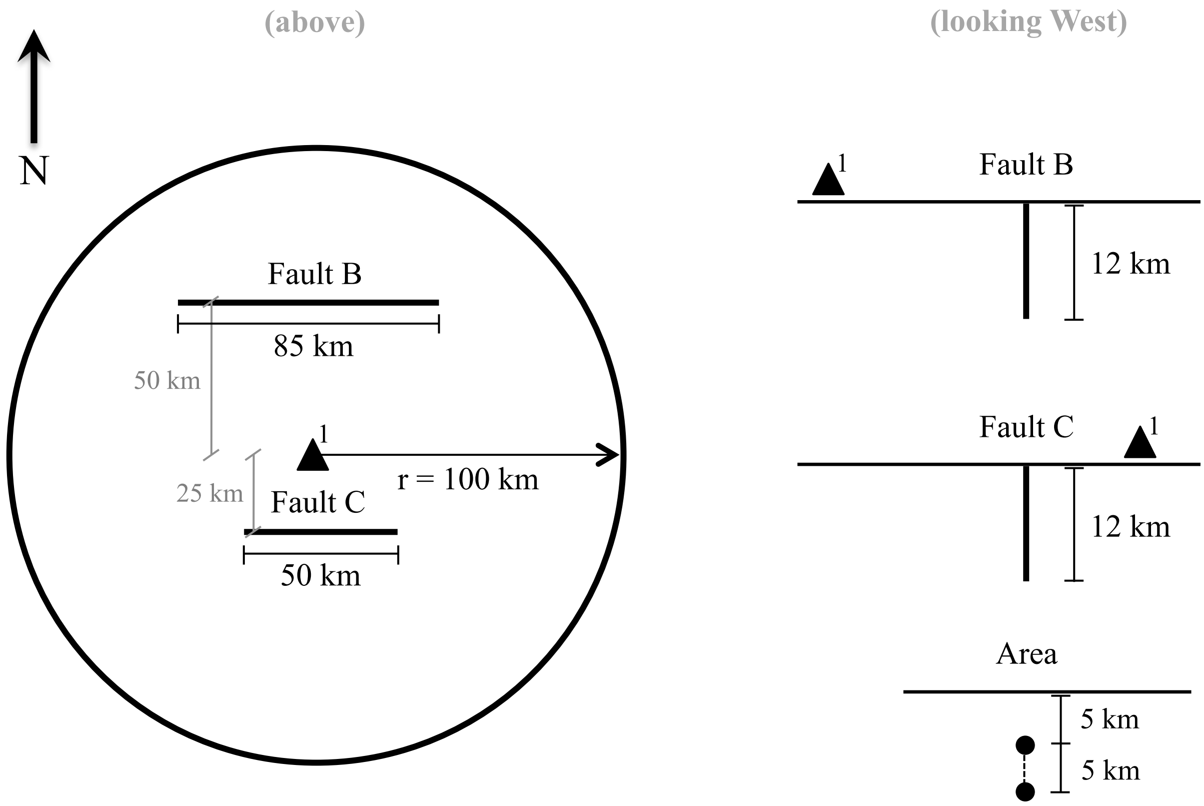
- **a.** PGA 0.05g
- **b.** The PGA corresponding to a hazard (annual exceedance probability) of 0.001.
- **c.** PGA 0.35g

Tables: Provide a table for each PGA with distance ($R_{rup}$), magnitude, and epsilon* bins. Distance bins should each be ~~10~~ 20 km, starting at 0 km and ending at 100 km with an extra bin for >100 km (~~5~~ 6 bins). Magnitude bins should each be 0.1 M, starting at M 5.0 and ending at M 7.0 (20 bins). The epsilon* bins should be defined as <-1, -1 to 0, 0 to 1, 1 to 2, >2 (5 bins).

| Distance (km) | Magnitude | Epsilon* | Deaggregation Results |
|---|---|---|---|
| ~~0-10~~ 0-20 | 5.0-5.1 | <-1 | |
| ~~10-20~~ 20-40 | 5.0-5.1 | <-1 | |
| ↓ | 5.0-5.1 | <-1 | |
| ~~80-90~~ 80-100 | 5.0-5.1 | <-1 | |
| ~~90-100~~ >100 | 5.0-5.1 | <-1 | |
| | | | |
| ~~0-10~~ 0-20 | 5.1-5.2 | <-1 | |
| ~~10-20~~ 20-40 | 5.1-5.2 | <-1 | |
| ↓ | 5.1-5.2 | <-1 | |
| ~~80-90~~ 80-100 | 5.1-5.2 | <-1 | |
| ~~90-100~~ >100 | 5.1-5.2 | <-1 | |
| | ↓ | | |
| | 6.9-7.0 | <-1 | |
| | | ↓ | |
| | | >2 | |

Averages: Provide the mean values $\overline{M}$, $\overline{R}$, $\overline{\varepsilon^*}$ for each PGA at Site 1. For this test case the distance term is $R_{rup}$ (further explanation on deaggregation terms attached).

**Figure 2.1 – Multiple sources, Deaggregation**




Note: figures not to scale

**Fault coordinates**

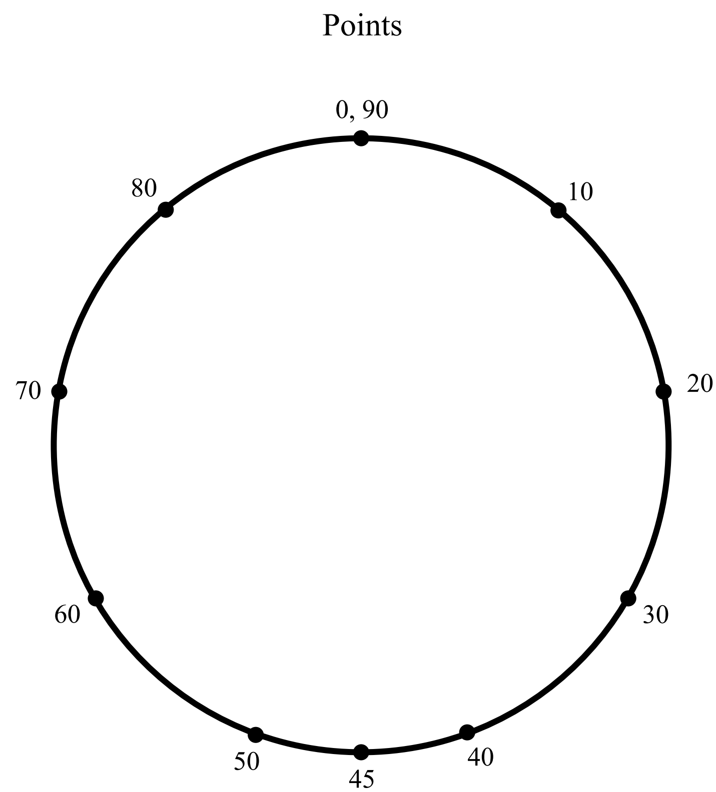
| Latitude | Longitude | Comment |
|---|---|---|
| 0.44966 | -65.38222 | West end of Fault B |
| 0.44966 | -64.61778 | East end of Fault B |
| -0.22483 | -65.22484 | West end of Fault C |
| -0.22483 | -64.77516 | East end of Fault C |

**Site coordinates**

| Site | Latitude | Longitude | Comment |
|---|---|---|---|
| 1 | 0.00000 | -65.00000 | In center of area source |

**Area source coordinates**

| Point | Latitude | Longitude |
|---|---|---|
| 0 | 0.8993 | -65.0000 |
| 1 | 0.8971 | -64.9373 |
| 2 | 0.8906 | -64.8748 |
| 3 | 0.8797 | -64.8130 |
| 4 | 0.8645 | -64.7521 |
| 5 | 0.8451 | -64.6924 |
| 6 | 0.8216 | -64.6342 |
| 7 | 0.7940 | -64.5778 |
| 8 | 0.7627 | -64.5234 |
| 9 | 0.7276 | -64.4714 |
| 10 | 0.6899 | -64.4219 |
| 11 | 0.6469 | -64.3753 |
| 12 | 0.6017 | -64.3316 |
| 13 | 0.5537 | -64.2913 |
| 14 | 0.5029 | -64.2544 |
| 15 | 0.4496 | -64.2211 |
| 16 | 0.3942 | -64.1917 |
| 17 | 0.3369 | -64.1662 |
| 18 | 0.2779 | -64.1447 |
| 19 | 0.2176 | -64.1274 |
| 20 | 0.1562 | -64.1143 |
| 21 | 0.0940 | -64.1056 |
| 22 | 0.0314 | -64.1012 |
| 23 | -0.0314 | -64.1012 |
| 24 | -0.0940 | -64.1056 |
| 25 | -0.1562 | -64.1143 |
| 26 | -0.2176 | -64.1274 |
| 27 | -0.2779 | -64.1447 |



**Area source coordinates**

| Point | Latitude | Longitude |
|---|---|---|
| 28 | -0.3369 | -64.1662 |
| 29 | -0.3942 | -64.1917 |
| 30 | -0.4496 | -64.2211 |
| 31 | -0.5029 | -64.2544 |
| 32 | -0.5537 | -64.2913 |
| 33 | -0.6017 | -64.3316 |
| 34 | -0.6469 | -64.3753 |
| 35 | -0.6889 | -64.4219 |
| 36 | -0.7276 | -64.4714 |
| 37 | -0.7627 | -64.5234 |
| 38 | -0.7940 | -64.5778 |
| 39 | -0.8216 | -64.6342 |
| 40 | -0.8451 | -64.6924 |
| 41 | -0.8645 | -64.7521 |
| 42 | -0.8797 | -64.8130 |
| 43 | -0.8906 | -64.8748 |
| 44 | -0.8971 | -64.9373 |
| 45 | -0.8993 | -65.0000 |
| 46 | -0.8971 | -65.0627 |
| 47 | -0.8906 | -65.1252 |
| 48 | -0.8797 | -65.1870 |
| 49 | -0.8645 | -65.2479 |
| 50 | -0.8451 | -65.3076 |
| 51 | -0.8216 | -65.3658 |
| 52 | -0.7940 | -65.4222 |
| 53 | -0.7627 | -65.4766 |
| 54 | -0.7276 | -65.5286 |

| Point | Latitude | Longitude |
|---|---|---|
| 55 | -0.6889 | -65.5781 |
| 56 | -0.6469 | -65.6247 |
| 57 | -0.6017 | -65.6684 |
| 58 | -0.5537 | -65.7087 |
| 59 | -0.5029 | -65.7456 |
| 60 | -0.4496 | -65.7789 |
| 61 | -0.3942 | -65.8083 |
| 62 | -0.3369 | -65.8338 |
| 63 | -0.2779 | -65.8553 |
| 64 | -0.2176 | -65.8726 |
| 65 | -0.1562 | -65.8857 |
| 66 | -0.0940 | -65.8944 |
| 67 | -0.0314 | -65.8988 |
| 68 | 0.0314 | -65.8988 |
| 69 | 0.0940 | -65.8944 |
| 70 | 0.1562 | -65.8857 |
| 71 | 0.2176 | -65.8726 |
| 72 | 0.2779 | -65.8553 |
| 73 | 0.3369 | -65.8338 |
| 74 | 0.3942 | -65.8083 |
| 75 | 0.4496 | -65.7789 |
| 76 | 0.5029 | -65.7456 |
| 77 | 0.5537 | -65.7087 |
| 78 | 0.6017 | -65.6684 |
| 79 | 0.6469 | -65.6247 |
| 80 | 0.6889 | -65.5781 |
| 81 | 0.7276 | -65.5286 |

**Area source coordinates**

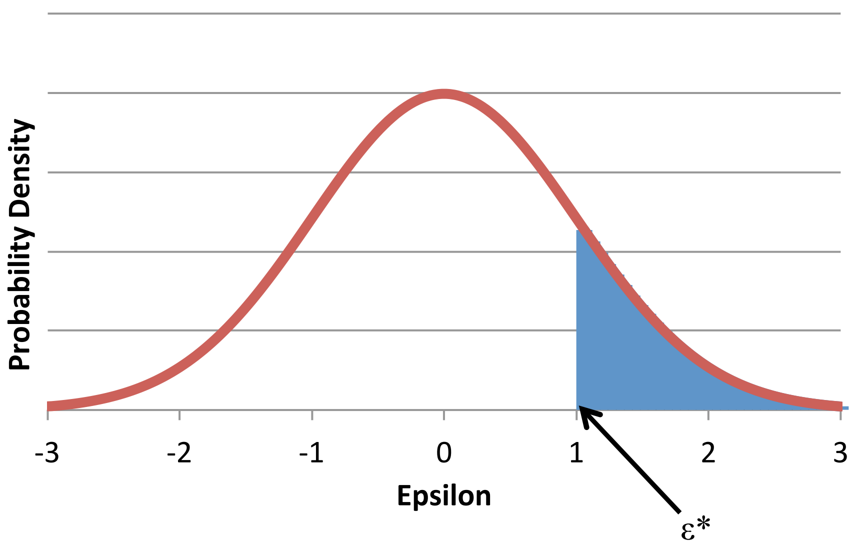
| Point | Latitude | Longitude |
|---|---|---|
| 82 | 0.7627 | -65.4766 |
| 83 | 0.7940 | -65.4222 |
| 84 | 0.8216 | -65.3658 |
| 85 | 0.8451 | -65.3076 |
| 86 | 0.8645 | -65.2479 |
| 87 | 0.8797 | -65.1870 |
| 88 | 0.8906 | -65.1252 |
| 89 | 0.8971 | -65.0627 |
| 90 | 0.8993 | -65.0000 |

(Point 90 is a duplicate of point 0 and closes the circle)

**Case 2.1 Explanation – Multiple sources, Deaggregation**

Epsilon*

The epsilon corresponding to a particular ground motion would just be epsilon, but in PSHA we calculate the probability of EXCEEDING a particular ground motion. Therefore, the epsilon (which we've denoted epsilon*) actually corresponds to any ground motion greater than the ground motion specified, and can be thought of as the minimum epsilon needed to exceed that ground motion, as illustrated below:




Mean deagreggation values

Starting with the typical equation for a seismic hazard analysis, the total annual rate of events, ν, with spectral accelerations, *Sa*, that exceed a specified value, *z*, is given by:

$$\nu(Sa > z) = \sum_{i=1}^{Nsource} N_i(M_{\min}) \int_{r=0}^{\infty} \int_{m=M_{\min}}^{M_{\max}} f_{m_i}(m) f_{r_i}(r) P(Sa > z | m, r)\, dr\, dm$$

Where $N_i(M_{min})$ is the annual rate of earthquakes with magnitude greater than or equal to $M_{min}$, $r$ is the distance from the source to the site, $m$ is earthquake magnitude, $f_m(m)$ and $f_r(r)$ are probability density functions for magnitude and distance, and $P(Sa>z|m,r)$ is the conditional probability of observing a spectral acceleration, *Sa*, greater than $z$ for a given earthquake magnitude and distance.

The mean magnitude and mean distance values are the weighted averages with the weights given by the deaggregation. More specifically, the mean is the conditional mean given the exceedance of the specified ground motion. The equation for the mean magnitude is given by multiplying the magnitude inside the hazard integral (in other words multiplying the magnitude by the marginal hazard). Likewise, the equation for the mean distance is given by multiplying the distance inside the hazard integral:

**Case 2.1 Explanation – Multiple sources, Deaggregation (continued)**

$$\overline{M} = \frac{\sum_{i=1}^{NSource} N_i(M_{\min}) \int_{r=0}^{\infty} \int_{m=M_{\min}}^{M_{\max}} M f_{m_i}(m) f_{r_i}(r) P(Sa > z \mid m,r)\, dr\, dm}{\nu(Sa > z)}$$

$$\overline{R} = \frac{\sum_{i=1}^{NSource} N_i(M_{\min}) \int_{r=0}^{\infty} \int_{m=M_{\min}}^{M_{\max}} R f_{m_i}(m) f_{r_i}(r) P(Sa > z \mid m,r)\, dr\, dm}{\nu(Sa > z)}$$

The mean epsilon* is computed in exactly the same way, but for clarity, let's first rewrite the seismic hazard equation to replace the $P(Sa > z \mid m,r)$ term with an equation that explicitly shows ε*:

$$P(Sa > z \mid m,r) = 1 - \Phi(\varepsilon^*)$$

$$\nu(Sa > z) = \sum_{i=1}^{Nsource} N_i(M_{\min}) \int_{r=0}^{\infty} \int_{m=M_{\min}}^{M_{\max}} f_{m_i}(m) f_{r_i}(r) [1 - \Phi(\varepsilon^*)]\, dr\, dm$$

Where Φ( ) is the standard normal cumulative distribution function, and ε* is computed by:

$$\varepsilon^* = \frac{\ln z - \overline{\ln Sa}}{\sigma_{\ln Sa}}$$

Now the equation for the mean epsilon* is given by multiplying the epsilon* inside the hazard integral:

$$\overline{\varepsilon^*} = \frac{\sum_{i=1}^{NSource} N_i(M_{\min}) \int_{r=0}^{\infty} \int_{m=M_{\min}}^{M_{\max}} \varepsilon^* f_{m_i}(m) f_{r_i}(r) [1 - \Phi(\varepsilon^*)]\, dr\, dm}{\nu(Sa > z)}$$

For an example of how to compute the mean magnitude, mean distance, and mean epsilon*, please see the excel file, "Deaggregation."

**Case 2.2 – NGA West 2 Ground Motion Models**

Description: Calculate the hazard for the six sites shown in Figure 2.2 due to Fault 3 using the specified NGA West 2 ground motion models.

Magnitude Density Function: truncated exponential, $M_{min}$ = 5.0, $M_{max}$= 7.0, b-value = 0.9

Source: Fault 3, L = 85 km, fault plane depths = 0-12 km, strike-slip, dip 90°, slip rate = 2 mm/yr

Ground motion models:

~~**a1.** Abrahamson, Silva, and Kamai 2014, σ = 0~~
**a2.** Abrahamson, Silva, and Kamai 2014, σ untruncated
~~**b1.** Boore, Stewart, Seyhan, and Atkinson 2014, σ = 0~~
**b2.** Boore, Stewart, Seyhan, and Atkinson 2014, σ untruncated
~~**c1.** Campbell and Bozorgnia 2014, σ = 0~~
**c2.** Campbell and Bozorgnia 2014, σ untruncated
~~**d1.** Chiou and Youngs 2014, σ = 0~~
**d2.** Chiou and Youngs 2014, σ untruncated

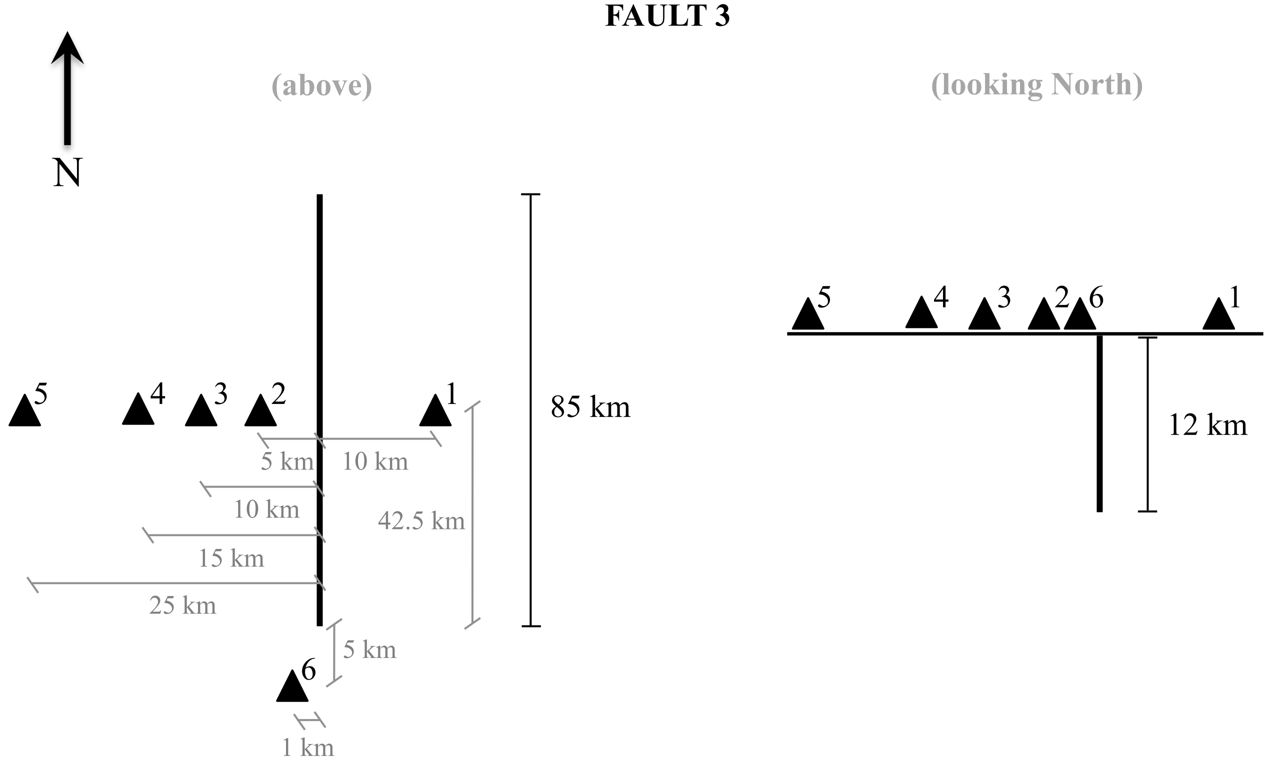
Damping ratio = 5%
$V_{S30}$ = 760 m/s
$V_{S30}$ is measured
$Z_{1.0}$ = 0.048 km
$Z_{2.5}$ = 0.607 km
Region = California

Rupture plane:

Log (A) = **M** – 4 $\sigma_A = 0$
Log (W) = (0.5***M**) – 2.15 $\sigma_W = 0$
Log (L) = (0.5***M**) – 1.85 $\sigma_L = 0$
Aspect Ratio (L/W) = 2
Maintain the aspect ratio defined until maximum width is reached, then increase length (conservation of area at the expense of aspect ratio)
Uniform distribution along strike and down dip
Use a hypocenter depth location in the geometric center of the rupture plane

**Figure 2.2 – NGA West 2 Ground Motion Models**

**FAULT 3**




Note: figures not to scale

**Fault coordinates**

| Latitude | Longitude | Comment |
|---|---|---|
| 0.38221 | -65.00000 | North end of fault |
| -0.38221 | -65.00000 | South end of fault |

**Site coordinates**

| Site | Latitude | Longitude | Comment |
|---|---|---|---|
| 1 | 0.00000 | -64.91005 | 10 km east of fault, at midpoint along strike |
| 2 | 0.00000 | -65.04497 | 5 km west of fault, at midpoint along strike |
| 3 | 0.00000 | -65.08995 | 10 km west of fault, at midpoint along strike |
| 4 | 0.00000 | -65.13490 | 15 km west of fault, at midpoint along strike |
| 5 | 0.00000 | -65.22483 | 25 km west of fault, at midpoint along strike |
| 6 | -0.42718 | -65.00900 | 5 km south of southern end, 1 km west |

**Case 2.3 – Hanging Wall**

Description: Calculate the hazard for the six sites shown in Figure 2.3 due to a single-magnitude event on Fault 4 using the specified NGA West 2 ground motion models.

Magnitude Density Function: delta function at **M** 7.0

Source: Fault 4, L = 85 km, fault plane depths = 1-12 km, reverse, dip 45°, slip rate = 2 mm/yr

Ground motion models:

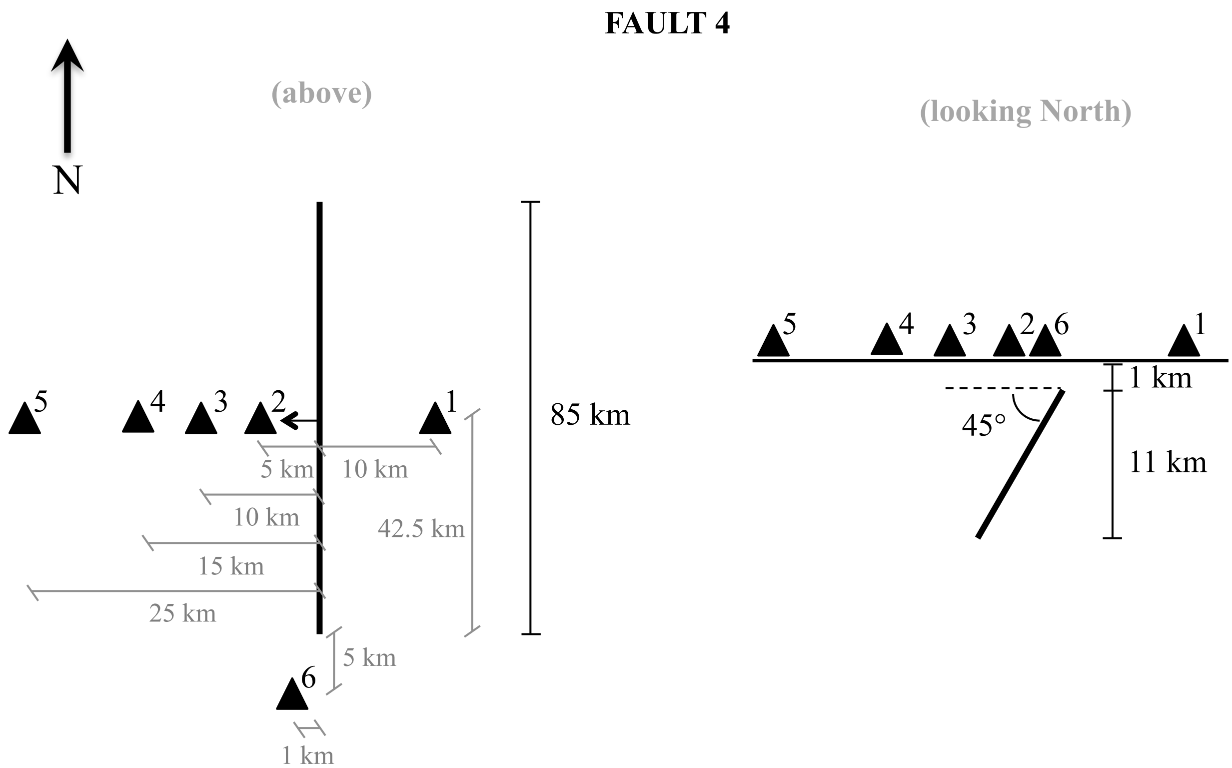
- **a.** Abrahamson, Silva, and Kamai 2014, $\sigma = 0$
- **b.** Boore, Stewart, Seyhan, and Atkinson 2014, $\sigma = 0$
- **c.** Campbell and Bozorgnia 2014, $\sigma = 0$
- **d.** Chiou and Youngs 2014, $\sigma = 0$

Damping ratio = 5%
$V_{S30}$ = 760 m/s
$V_{S30}$ is measured
$Z_{1.0}$ = 0.048 km
$Z_{2.5}$ = 0.607 km
Region = California

Rupture plane:

Log (A) = **M** – 4 $\sigma_A = 0$
Log (W) = (0.5***M**) – 2.15 $\sigma_W = 0$
Log (L) = (0.5***M**) – 1.85 $\sigma_L = 0$
Aspect Ratio (L/W) = 2
Maintain the aspect ratio defined until maximum width is reached, then increase length (conservation of area at the expense of aspect ratio)
Uniform distribution along strike and down dip
Use a hypocenter depth location in the geometric center of the rupture plane

**Figure 2.3 – Hanging Wall**




Note: figures not to scale

**Fault coordinates**

| Latitude | Longitude | Comment |
|---|---|---|
| 0.38221 | -65.00000 | North end of fault |
| -0.38221 | -65.00000 | South end of fault |

**Site coordinates**

| Site | Latitude | Longitude | Comment |
|---|---|---|---|
| 1 | 0.00000 | -64.91005 | 10 km east of fault, at midpoint along strike (FW) |
| 2 | 0.00000 | -65.04497 | 5 km west of fault, at midpoint along strike (HW) |
| 3 | 0.00000 | -65.08995 | 10 km west of fault, at midpoint along strike (HW) |
| 4 | 0.00000 | -65.13490 | 15 km west of fault, at midpoint along strike (HW) |
| 5 | 0.00000 | -65.22483 | 25 km west of fault, at midpoint along strike (HW) |
| 6 | -0.42718 | -65.00900 | 5 km south of southern end, 1 km west (HW side) |

**Case 2.4 – Uniform, Triangular distribution of hypocenter locations down dip**

**a. Uniform distribution down dip**

Description: Calculate the hazard for the site shown in Figure 2.4 due to a single-magnitude event on Fault 5 using the specified NGA West 2 ground motion models. (The purpose of this test is to have a comparison for the triangular distribution in part b).

Magnitude Density Function: delta function at **M** 6.0

Source: Fault 5, L = 25 km, fault plane depths = 0-30 km, strike-slip, dip 90°, slip rate = 2 mm/yr

Ground motion models: Chiou and Youngs 2014, $\sigma = 0$

Damping ratio = 5%
$V_{S30}$ = 760 m/s
$V_{S30}$ is measured
$Z_{1.0}$ = 0.048 km
$Z_{2.5}$ = 0.607 km
Region = California

Rupture plane:

Log (A) = **M** – 4 $\sigma_A = 0$
Log (W) = (0.5***M**) – 2.15 $\sigma_W = 0$
Log (L) = (0.5***M**) – 1.85 $\sigma_L = 0$
Aspect Ratio (L/W) = 2
Maintain the aspect ratio defined until maximum width is reached, then increase length (conservation of area at the expense of aspect ratio)
Uniform distribution along strike and down dip
Use a hypocenter depth location in the geometric center of the rupture plane

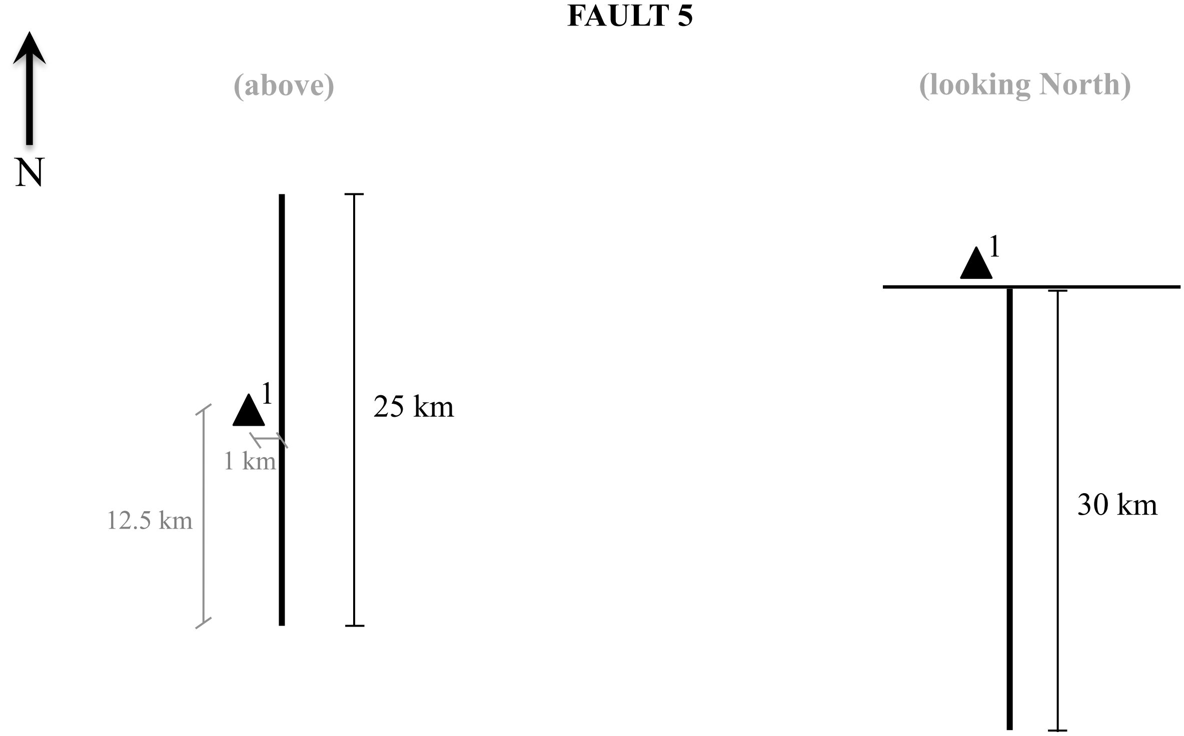
**b. Triangular distribution of hypocenter locations down dip**

Description: Use the same specifications above, but replace the uniform distribution down dip with a triangular distribution of hypocenter locations down dip (further explanation on triangular distribution attached).

Triangular distribution of hypocenter locations down dip (0 km, 10 km, 30 km)

**Figure 2.4 - Uniform, Triangular distribution of hypocenter locations down dip**

**FAULT 5**




Note: figures not to scale

**Fault coordinates**

| Latitude | Longitude | Comment |
|---|---|---|
| 0.11240 | -65.00000 | North end of fault |
| -0.11240 | -65.00000 | South end of fault |

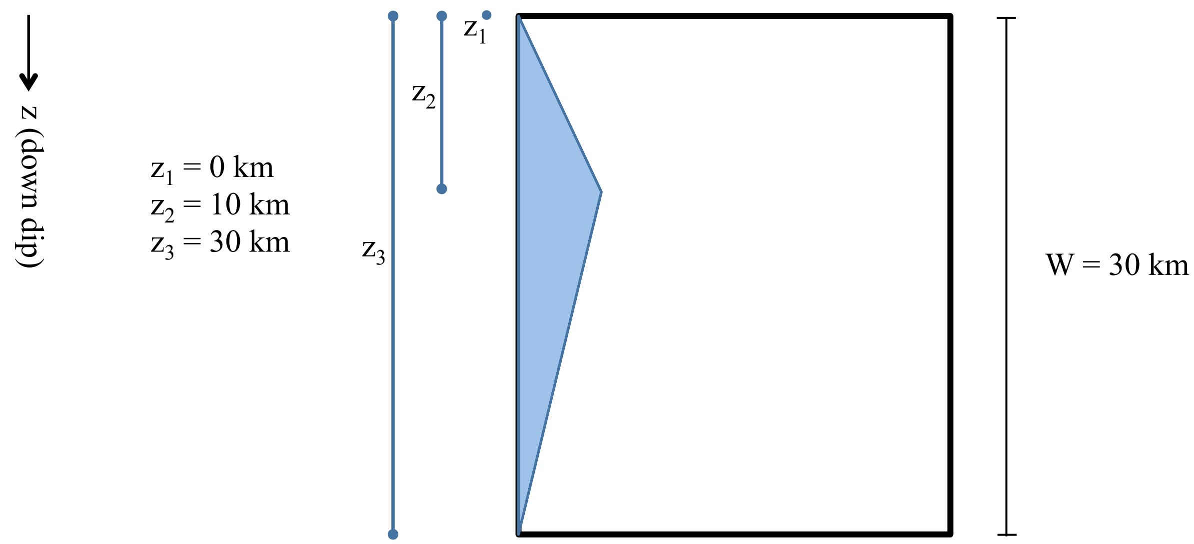
**Site coordinates**

| Site | Latitude | Longitude | Comment |
|---|---|---|---|
| 1 | 0.00000 | -65.00900 | 1 km west of fault, at midpoint along strike |

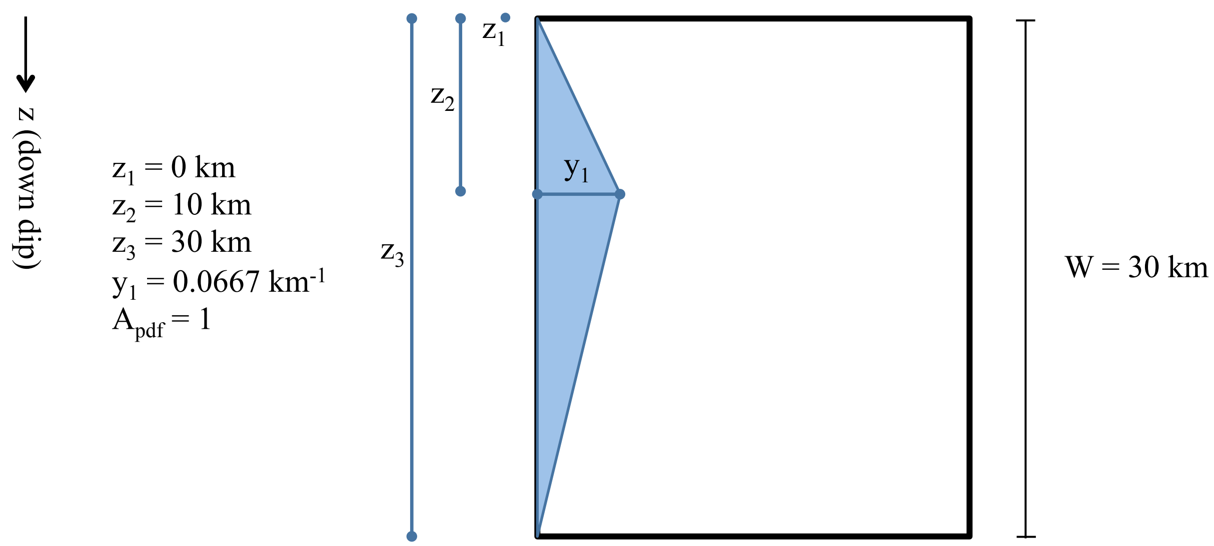
**Case 2.4 Explanation – Triangular distribution of hypocenter locations down dip**

The location of the rupture plane on the fault in the down dip direction is based on hypocenter observations. When the source characterization experts specify a triangular distribution they are basing that on hypocenter locations on the fault plane. Therefore, it is not simply the rupture plane that follows the triangular distribution, but specifically the hypocenter location on the rupture plane.

We specified the distribution of the hypocenter down dip as a triangle (0 km, 10 km, 30 km). Looking at the fault plane, the triangle looks like this:

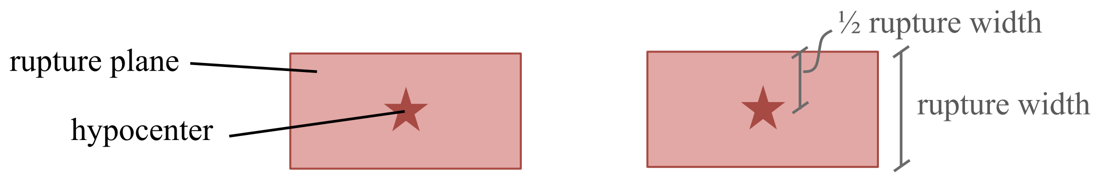



The area of the triangle (which is a probability density function) must be equal to 1. With the given specifications, the value of $y_1$ is 1/15 km$^{-1}$ or 0.0667 km$^{-1}$ (note $y_1$ is not drawn to scale).

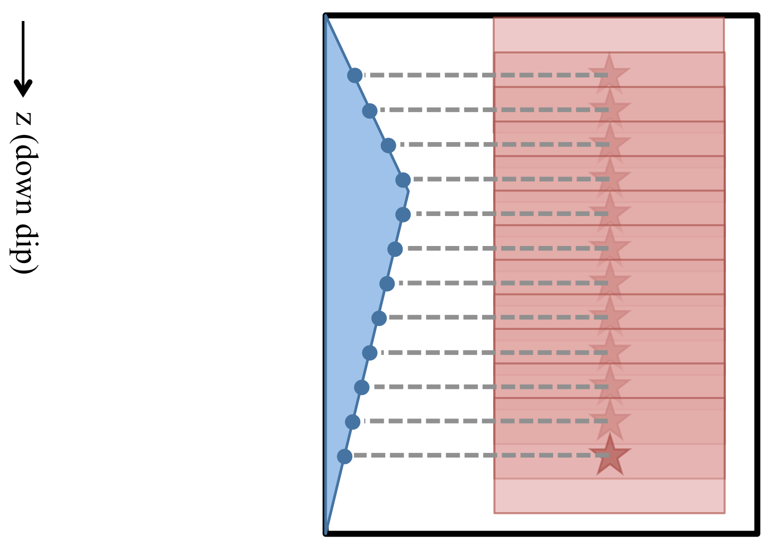


**Case 2.4 Explanation – Triangular distribution of hypocenter locations down dip (continued)**

The location of the hypocenter on the rupture plane itself can be modeled with a probability density function. For simplicity, we're going to use a delta function, with the hypocenter location always in the center of the rupture plane, or at one half the rupture plane width:




Now we move the rupture plane down the dip of the fault. Because we don't allow ruptures to go into the air, the first rupture plane's top edge is at a depth of zero, and it's hypocenter is at a depth of ½ the rupture width. As we move the rupture plane down the dip of the fault, the location of the hypocenter corresponds to a probability density on the triangular distribution. Below I've used a relatively large step size for illustrative purposes only. For the validation exercise you should use a step size as small as necessary to reach stable results:




In the above illustration the blue dots represent the portion of the pdf that we sampled. Because the entire pdf was not sampled, our probabilities will not sum to one and we must renormalize the pdf so that the probabilities sum to one. The pdf should be renormalized based on the hypocenter locations that were sampled.

**Case 2.5 – Upper Tails, Mixture Model**

**a. Upper Tails**

Description: Calculate the hazard for the site shown in Figure 2.5 due to a single-magnitude event on Fault ~~5~~ 6 using the specified NGA West 2 ground motion model. The purpose of this scenario is to test the ability to model a normal distribution out to high epsilon values. Because we are interested in the upper tails of the distribution, we need to change the PGA values. Please provide mean hazard results (probability of exceedance) for peak horizontal acceleration (PGA) defined at 0.001, 0.01, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.25, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 6.0, and 7.0 g.

Magnitude Density Function: delta function at **M** 6.0

Source: Fault 5, L = 25 km, fault plane depths = 0-12 km, strike-slip, dip 90°, slip rate = 2 mm/yr

Ground motion model: Chiou and Youngs 2014, use a fixed $\sigma = 0.65$, untruncated

Damping ratio = 5%
$V_{S30} = 760$ m/s
$V_{S30}$ is measured
$Z_{1.0} = 0.048$ km
$Z_{2.5} = 0.607$ km
Region = California

Rupture plane:

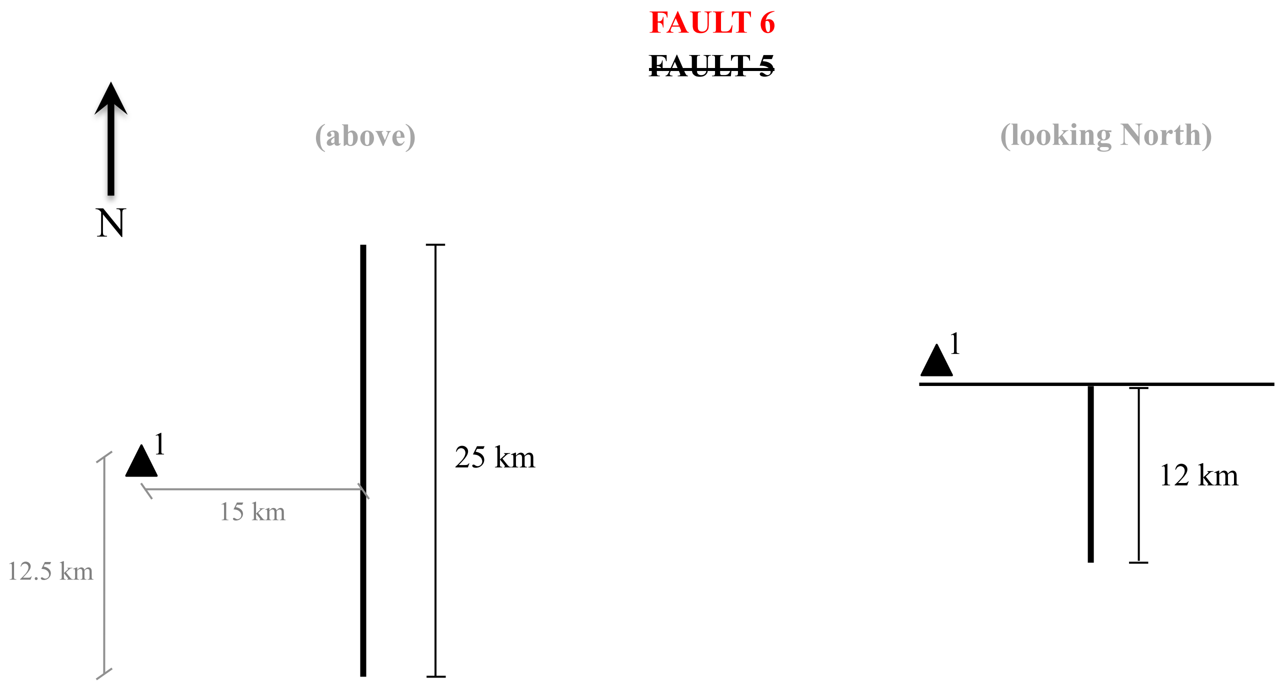
$\text{Log}(A) = \mathbf{M} - 4$ $\quad \sigma_A = 0$
$\text{Log}(W) = (0.5*\mathbf{M}) - 2.15$ $\quad \sigma_W = 0$
$\text{Log}(L) = (0.5*\mathbf{M}) - 1.85$ $\quad \sigma_L = 0$
Aspect Ratio (L/W) = 2
Maintain the aspect ratio defined until maximum width is reached, then increase length (conservation of area at the expense of aspect ratio)
Uniform distribution along strike and down dip
Use a hypocenter depth location in the geometric center of the rupture plane

**b. Mixture Model**

Description: Use the same specifications above, but adjust the sigma (still fixed $\sigma = 0.65$) with the following mixture model specifications (further explanation on mixture models attached):

Mixture Model: two normal distributions
Distribution 1: weight, $w_{Mix1} = 0.5$, sigma, $\sigma_{Mix1} = 1.2\,\sigma$
Distribution 2: weight, $w_{Mix2} = 0.5$, sigma, $\sigma_{Mix2} = 0.8\,\sigma$

**Figure 2.5 – Mixture Model**




Note: figures not to scale

**Fault coordinates**

| Latitude | Longitude | Comment |
|---|---|---|
| 0.11240 | -65.00000 | North end of fault |
| -0.11240 | -65.00000 | South end of fault |

**Site coordinates**

| Site | Latitude | Longitude | Comment |
|---|---|---|---|
| 1 | 0.00000 | -65.13490 | 15 km west of fault, at midpoint along strike |

**Case 2.5 Explanation – Mixture Model**

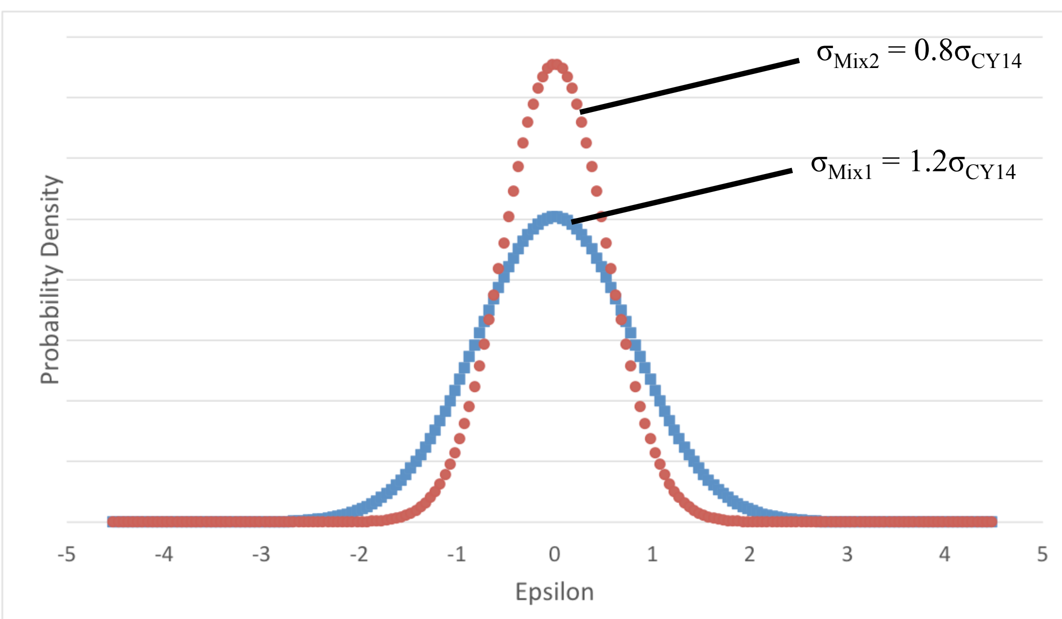
*Note that this explanation uses the CY14 sigma, but the test case now specifies a fixed sigma value.

A mixture model is a combination of two distributions. In our case, the ground motion mixture model is a combination of two normal distributions. The mixture model is used because the earthquake data do not exactly fit the typical normal distribution at high epsilons. For the case of shallow crustal earthquakes, the normal distribution is a good model for ground motions up until the tails of the distribution (epsilons of about 2.5) and then the data start to deviate from a normal distribution. If you plot the observed shallow crustal earthquake data density against the assumed normal distribution from the NGA West 2 GMPEs you will start to see that the data show heavier tails than the predicted values (meaning that observations show a higher probability of extremes than provided by the normal distribution). For most projects these high epsilon values are beyond the range of interest, but for projects like Yucca Mountain this misfit can be important.

The typical approach to represent a heavy-tailed distribution for ground motions is to use a mixture model consisting of a weighted mixture of two normal distributions, one with a larger variance, and one with a smaller variance. The standard deviation for each distribution is specified as a ratio of the standard deviation of the ground motion model. The two mixtures are combined by specifying a weight for each distribution. If you calibrate the mixture model using the CY14 residuals, the standard deviation ratios are 1.2 and 0.8 and the weight for each distribution is 50%. For this mixture model the conditional probability of exceeding a ground motion level Z is given by:

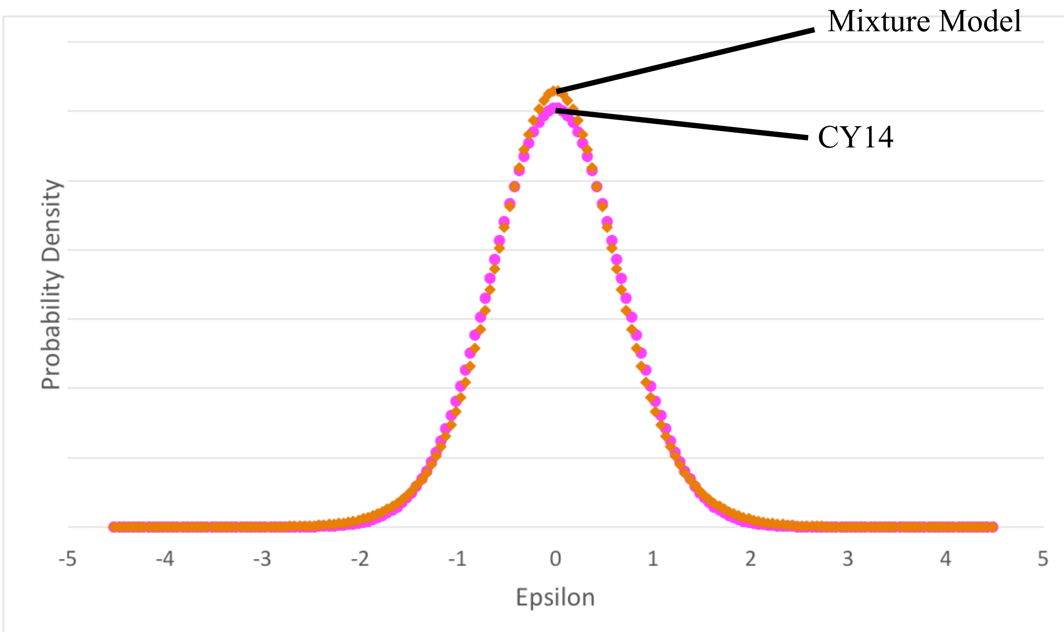
$$P(Z > z) = w_{Mix1}\left\{1 - \Phi\frac{z-\mu}{\sigma_{Mix1}}\right\} + w_{Mix2}\left\{1 - \Phi\frac{z-\mu}{\sigma_{Mix2}}\right\}$$

The two normal distributions with sigmas equal to $0.8\sigma_{CY14}$ and $1.2\sigma_{CY14}$ look like this:

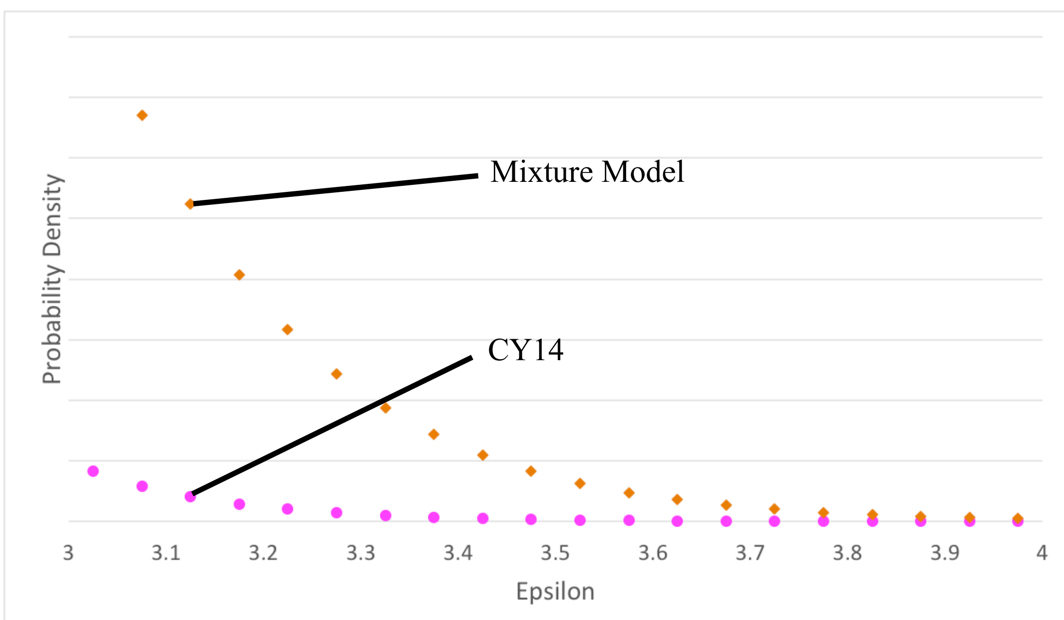


**Case 2.5 Explanation – Mixture Model (continued)**

Comparing the distribution for the CY14 ground motion model with the distribution for the mixture model shows that the distributions are nearly the same for the epsilon range -2.5 to 2.5, where the single normal distribution was a good fit for the data:

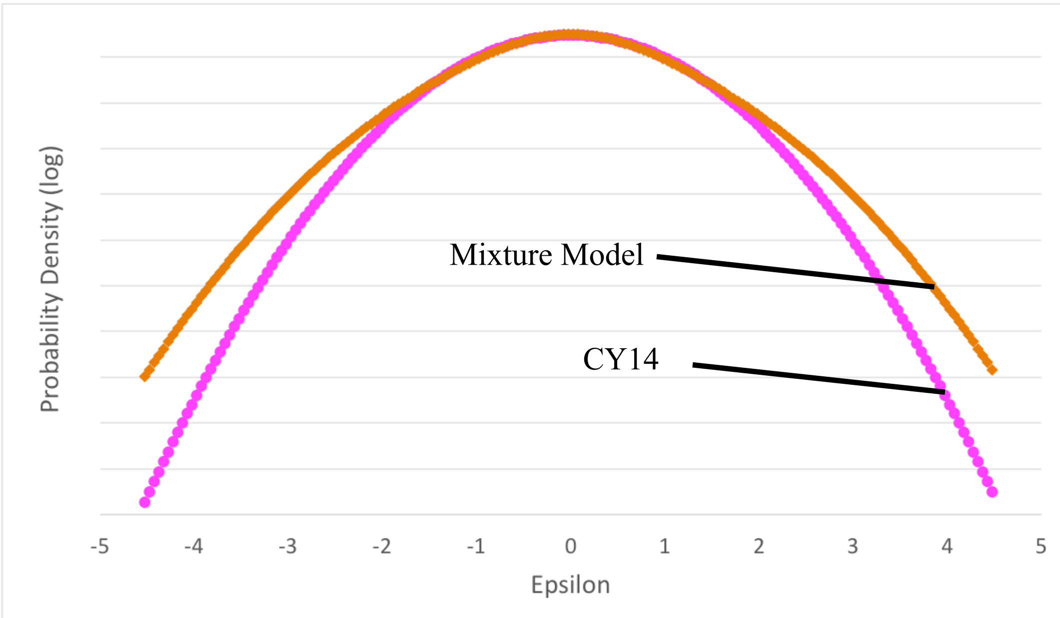



The above figure makes it difficult to see how the mixture model accounts for the heavier tails that were needed, but if we zoom in at the high epsilon range we can see the increased probability density at high epsilons:

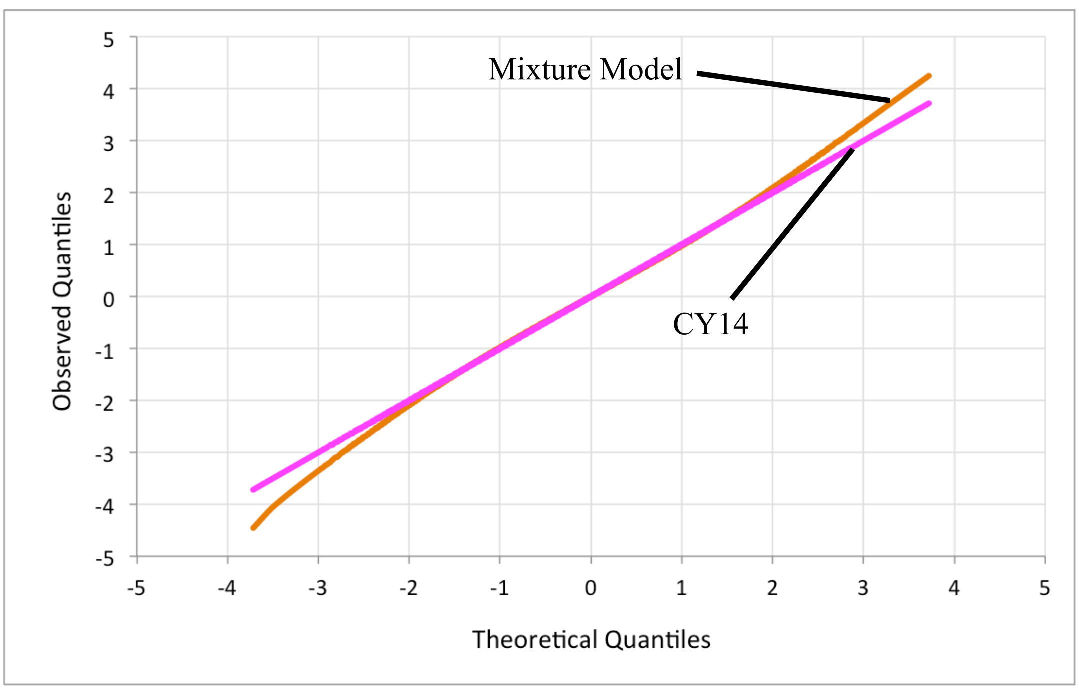


**Case 2.5 Explanation – Mixture Model (continued)**

In order to see the difference between the two distributions for the full range of epsilons we need to compare the distributions on a plot with the probability density (y-axis) on a log scale. This plot shows the heavier tails of the mixture model:




We could also look at the distributions on a Q-Q plot, where we consider the CY14 model the theoretical distribution and the mixture model the observed:




*Reference for Mixture Model explanation: Southwestern U.S. Ground Motion Characterization SSHAC Report, draft.*